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THEORETICAL MODEL OF NONEQUILIBRIUM OF EXTRACTION

OF A GAS DISSOLVED IN A FLUID DURING PRESSURE

FLUCTUATIONS IN A FLOW

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An engineering model and a method of computing nonequilibrium extraction of a gas dissolved in a fluid during pressure fluctuations in a dispersely-nucleate gas-liquid flow that differs from a known flow by taking account of the turbulent nature of the relative phase motion and a more accurate determination of the gas bubble radius in an equivalent flow. The results of computing the magnitude of the additional gas-extraction during pressure fluctuations in a pipeline are compared with experimental data obtained earlier in water and carbon dioxide that verifies the reliability of the developed theoretical model.

### INTRODUCTION

Low-frequency pressure fluctuations due either to vibration loaded hydraulic systems or to cavitation self-scillations in their elements occur in their hydraulic systems during the operation of the majority of power plants.

As experimental investigations [1] showed, the pressure fluctuations in a gas-liquid flow cause additional gas extraction from the fluid with respect to the stationary case, that can result in a cavitation collapse of the supply pump operation. The reason causing additional gas extraction is the periodic change in the surface area of the phase separation in the flow. Under the action of pressure fluctuations on a gas-fluid flow the process of gas extraction alternates with the dissolution process. However, the mass transfer surface area will be greater during pressure diminution than during magnification. Consequently, more gas is extracted during one fluctuation period than is dissolved.

A theoretical analysis of this phenomenon, called rectified gas diffusion, was first performed in the Harvard Acoustic Research Laboratory and its results are represented in [2, 3]. Later more modern theoretical models [4-6] were developed, however, they are all based on a spherically-symmetric formulation of the gas diffusion problem without taking account of the relative motion of the gas bubbles and the fluid. At the same time a substantial relative motion of the gas and liquid phases occurs at the same time in power plant mainline supply systems. E. V. Vengerskii [7], who developed an engineering model and method during pressure fluctuations in a gas-liquid flow, performed a theoretical analysis of the rectified gas diffusion in this case.

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Experimental investigations [1] performed on water with carbon dioxide gas qualitatively confirmed the theoretical deduction of [7], however, the magnitude of the additional gas extraction turned out to be approximately two times less than the computed value. Analysis showed that the main reason for the discrepancy is the disadvantages of the theoretical model [7]. In particular, the turbulent nature of the relative phase motion is not taken into account in [7] and the bubble radius in an equivalent monodisperse flow is incorrectly determined in the mathematical description of the mass transfer. In connection with the disadvantages noted, the question of producing a more perfect theoretical model of the phenomenon is urgent.

# THEORETICAL MODEL

The following physical pattern of the process was taken to describe nonequilibrium mass transfer in a gas-liquid flow. A periodically varying pressure acts on a pipeline during the flow of a tubulent dispersely-nucleat gas-liquid flow. Depending on the instantaneous value of the pressure in the flow, alternation of the gas phase dissolution and extraction process occurs. The mass transfer intensity is here determined by convective diffusion in fluid layers adjoining the gas bubble. Since the flow is turbulent in nature, then the relative bubble and fluid motion is determined by the turbulent fluctuations of the fluid velocity. The formation and growth of gas bubbles both on the pipeline wall and in the fluid flow occur during gas extraction. As the gas phase accumulates on the pipeline wall, the gas bubbles go over into the flow. The magnitude of the separation radius of the bubbles is determined by the physical properties of the fluid and the flow velocity. The process of bubble breakdown and merger occurs in the flow. The magnitude of the limit radius of the gas bubbles during breakdown is determined by turbulent fluid velocity fluctuations.

The following assumptions were taken to simplify the mathematical description of mass transfer processes: 1) the gas bubble expansion and compression processes occur isothermally while the gas phase filling the bubbles is an ideal gas; 2) the pressure in the gas bubble equals the pressure of the surrounding fluid; 3) the action of the gravitational force on the gas-liquid flow is not taken into account; 4) the gas bubbles in the flow are spherical in shape and hemispherical on the wall. The physical pattern taken for the process constrains the domain of applicability of the theoretical model to the following range: the pressure fluctuation frequency is 0-100 Hz, the pressure fluctuation amplitude is a<0.8(p<sub>m</sub> - p<sub>s</sub>), the flow velocity is W = 1-10 m/sec, and the relative gas phase volume is  $\delta = 0-0.3$ .

It is expedient to replace the actual polydisperse flow by an equivalent monodisperse flow in which all the bubbles are of identical size for the mathematical description of the mass transfer processes. To this end, the following conditions for equivalence of the real and monodisperse flows were formulated

$$\delta_{\mathbf{e}} = \delta \text{and} S_{\mathbf{e}} = S, \tag{1}$$

that differ from those taken in [7] by equality of not only the relative volume but also the area of the gas phase surface. Taking account of the fact that the gas bubbles in the turbulent flow have a normal logarithmic size distribution law [8, 9], an expression is obtained on the basis of (1) for the bubble radius in the equivalent monodisperse flow

$$R_{\rm e} = \frac{3(R_{\rm max}^4 - R_{\rm min}^4)}{4(R_{\rm max}^3 - R_{\rm min}^3)}.$$
 (2)

The maximal gas bubble radius in the flow equals the limit value for breakdown and, according to the breakdown model [10] is determined by the relationship

$$R_{\rm max} = d^{2/5} \left(\frac{\sigma}{k_f \rho_{\rm f}}\right)^{3/5} \frac{1}{W^{6/5}} \left(\frac{\rho_{\rm f}}{\rho_{\rm g}}\right)^{1/5}.$$
 (3)

The minimal radius is determined by the dimensions of the seed bubbles and can be taken equal to zero.

To develop the mathematical model, we isolate a small fluid volume  $V_f$  in the flow (Fig. 1) and let us trace the change in its parameters during motion in the pipeline. The specific mass of the gas phase in the isolated volume equals the ratio between the gas mass and the fluid volume and is the following at the initial time  $m_0 = \delta_0 \rho_{r0}$ : The specific mass of the gas phase at an arbitrary time  $\tau_i$  is determined by the relationship



Fig. 1. Change in parameters of the isolated volume during its motion in the pipeline.

$$m = m_0 + \int_{\tau_0}^{\tau_i} \frac{dm}{d\tau} d\tau + \sum_{k=1}^n \Delta m_k,$$

where  $\Delta m_{\rm k}$  is the increment in the specific mass of the gas phase due to bubble separation from the wall.

The rate of change of the specific mass of the gas phase due to gas extraction in the flow is determined by a relationship obtained on the basis of the model of convective gas diffusion in bubbles [11]

$$\frac{dm}{d\tau} = \frac{3\delta}{R_{\mathbf{e}}} \left( c - \varkappa p_{\mathbf{r}} \right) \sqrt{\frac{2Du}{\pi R_{\mathbf{e}}}}$$

The increment in the specific mass of the gas phase in the flow due to bubble separation from the wall during one period of fluctuation equals

$$\Delta m_k = \int_{\frac{2\pi}{\omega}(k-1)}^{\frac{2\pi}{\omega}k} \frac{dm'}{d\tau} d\tau.$$

The rate of change of the specific mass of the gas phase on the pipeline surface is determined by the expression

$$\frac{dm'}{d\tau} = \frac{2\pi\rho_{go}(1+\delta)}{d\rho_{g}} 0,076 (c-\varkappa p_{g}) \frac{W^{1,17}R_d^{0,17}\lambda^{0,585}D^{0,5}}{v^{0,67}},$$

obtained on the basis of the data in [7]. Let us write the dissolved gas concentration in the form of the relationship

$$c_i = c_0 - \int_{\tau_0}^{\tau_i} \left( \frac{dm}{d\tau} + \frac{dm'}{d\tau} \right) d\tau.$$

The relative volume gas content in the isolated volume equals

$$\delta_i = m_i / \rho_{\mathbf{g}_i}.$$

The velocity of the isolated volume motion is

$$W = \delta \frac{Q_{\mathbf{f}}}{\pi d^2}.$$

Let us determine the relative velocity of gas bubble motion in the fluid by the expression

$$u = \frac{1}{k_f^{0.33}} \frac{(1 - \rho_{\mathbf{g}} \rho_{\mathbf{f}})^{0.5}}{(\rho_{\mathbf{g}} \rho_{\mathbf{f}} + 0.5)^{0.17}} \left(\frac{R_{\mathbf{e}}}{d}\right)^{0.33} W$$

obtained as a result of analyzing the action of turbulent fluid velocity fluctuations on the gas bubbles.

#### DISCUSSION

On the basis of the equations presented above, the sinusoidal law of pressure variation, the relationships (2) and (3) and the equation of state of an ideal gas, an algorithm for the numerical computation of the gas content of a two-phase flow based on applying the finite sum method, and an electronic computer computation program were compiled. By using this program computations were performed for conditions of the experiments for the research in [1].



Fig. 2. Dependence of the magnitude of the additional gas extraction  $\Delta \phi$  on the relative amplitude of the pressure fluctuation a.  $\Delta \phi$ , %

The computation results are represented in Fig. 2 in a coordinate system where the relative amplitude of the pressure fluctuations is plotted along the abscissa axis and the magnitude of the additional gas extraction, equal to the difference between the relative gas phase volumes in the flow with and without pressure fluctuations, is plotted along the ordinate. Curve 1 in the figure corresponds to the mathematical model developed and curve 2 to the model of the research in [7]. Presented in Fig. 2 in addition to the theoretical dependences are experimental values from [1].

Analysis of the data represented shows that the results of computations obtained by using the theoretical model developed are in good agreement with the experimental values. At the same time a computation utilizing the mathematical model in [7] yields strongly exaggerated results that is especially noticeable for the relative amplitude of the pressure fluctuations greater than 0.2.

### CONCLUSIONS

Pressure fluctuations in a gas-liquid flow result in additional extraction, as compared with the stationary case, of the gas phase from the dissolved state that grows as the amplitude of the pressure fluctuations increase.

The theoretical model developed describes the process of nonequilibrium extraction of a gas dissolved in a fluid well and can be used in the computation and design of power plants as well as other aggregates and technological processes where the flow of gas-liquid streams occurs under dynamic conditions.

## NOTATION

a, a, are the absolute and relative amplitude of the pressure fluctuations; c, is the dissolved gas concentration; D, is the coefficient of molecular diffusion; d, is the pipeline diameter; k, is the number of fluctuation periods; k<sub>f</sub>, is the bubble drag coefficient, m, m', is the specific mass of the gas phase in the stream and on the wall of the pipeline; p, is the pressure;  $p_s$ , is the saturated fluid vapor pressure; Q, is the mass flow rate; R, is the bubble radius;  $R_d$ , is the bubble separation separation radius; S, is the surface area of the gas phase in the isolated volume; u, is the relative velocity of gas bubble motion; V, is the volume; W, is the stream velocity; x, is the coordinate;  $\Delta \phi$ , is the Hengy solubility coefficient;  $\lambda$ , is the friction drag coefficient;  $\nu$ , is the kinematic viscosity coefficient;  $\rho$ , is the density;  $\sigma$ , is the coefficient of surface tension;  $\tau$ , is the time;  $\omega$ , is the circular frequency of the pressure fluctuations in the isolated volume. Subscripts: 0 is the zero-th time; i, is an arbitrary time; f, is fluid; g, is gas; min is minimal; max, is maximal; m, is mean; e, refers to the equivalent monodisperse flow.

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### APPROXIMATE DEPENDENCE BETWEEN THE THERMODYNAMIC

QUANTITIES OF A FLUID ON THE SATURATION LINE

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It is shown in the approximation of additivity of the molecule interaction potential in a fluid that the relationship uv ~ const is satisfied approximately on the saturation line. The deviation from experiment is 10-30% for the majority of fluids, and 5% for crystalline argon. The mentioned deviation of uv from experiment is explained by multiparticle molecule interaction not being taken into account in fluids, consequently, the role of the multiparticle interaction in condensed media can be assessed according to the magnitude of the deviation from experiment.

The following dependence between the internal heat of evaporation of a substance and its density is set up empirically in [1]

$$u = A \left(\frac{\rho_l - \rho_v}{\rho_h}\right)^n,\tag{1}$$

where  $\rho_{\ell}$ ,  $\rho_{v}$ ,  $\rho_{k}$  are, respectively, the fluid and vapor densities on the saturation line and at the critical point, A, n are constants characteristic for this substance. In conformity with the data in [1], the relationship (1) agrees with 2-3% accuracy with experiment for a number of substances with the exception of oxygen, where the exponent n varies in the narrow 1.1-1.2 band and in this sense is universal, which is, apparently, the "echo" law of corresponding states. The purpose of this paper is 1) to give a theoretical foundation of the empirical relationship (1) by starting from modern representations on the nature of fluids, and 2) to show that the investigation of the dependence of the heat of fluid evaporation u on its specific volume v on the satural line yields valuable information about the non-additive component of the molecule interaction potential in the fluid.

If the fluid properties are described by using the radial distribution function g(r), then the relationship

> $u \sim \frac{1}{v} \int_{0}^{\infty} \Phi(r) g(r) r^{2} dr,$ (2)

will be valid for the energy, where  $\Phi(\mathbf{r})$  is the pairwise potential of molecule interaction at a distance r. The main contribution to the integral of the relationship (2) yields a domain of distances  $r = r_1$  corresponding to the first maximum of the radial distribution function g(r). Indeed, for  $r < r_1$  the function g(r) drops quite rapidly to 0 and becomes a small interaction potential  $\Phi(r)$  at the distance  $r_2$  corresponding to the second coordinate sphere of g(r). Consequently, we can write

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